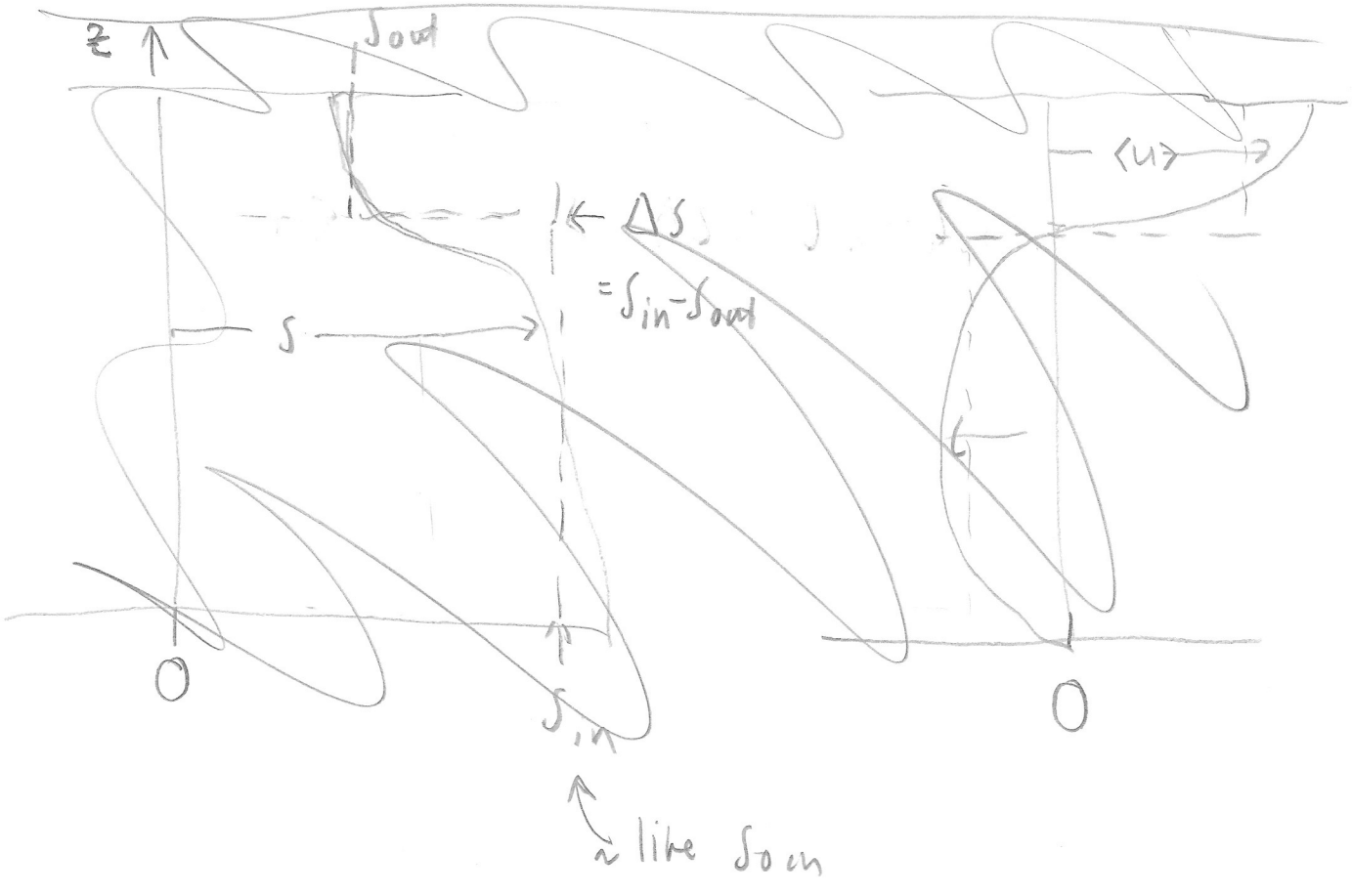
Knudsen Relations

8/13/2019 (1)

Consider an estuary

Approximate transport and salinity through mouth section (*) as two layers



Formally we calculate these by

① binning transport in salinity classes,
② taking tidal average, ③ and integrating
over incoming & outgoing salinity
ranges

("Total Exchange Flow": MacCready 2011 JPO)

Sign Convention

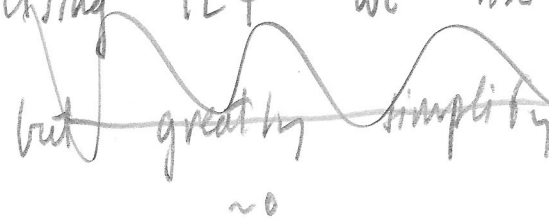
Q_{in} & Q_R : positive

Q_{out} : negative

Volume and salt budgets

Using TEF we lose all spatial structure,

but greatly simplify the fluxes.



~ 0

vol $\quad \cancel{\frac{dV}{dt}} = Q_{in} + Q_{out} + Q_R \qquad (*)$

ss $\quad \frac{d}{dt}\left\langle \int s\, dV \right\rangle = Q_{in} s_{in} + Q_{out} s_{out}$

$Q_{out} = -Q_{in} - Q_R$ from $(*)$

$s_{in} = s_{out} + \Delta s$

where $\Delta s \equiv s_{in} - s_{out}$

$$\Rightarrow \quad \frac{d}{dt}\left\langle \int s\, dV \right\rangle = \cancel{Q_{in} s_{out}} + Q_{in}\Delta s - \cancel{Q_{in} s_{out}} - Q_R s_{out}$$

$$\Rightarrow \quad \boxed{Q_{in} = \frac{s_{out}}{\Delta s} Q_R + \frac{1}{\Delta s}\frac{d}{dt}\left\langle \int s\, dV \right\rangle}$$

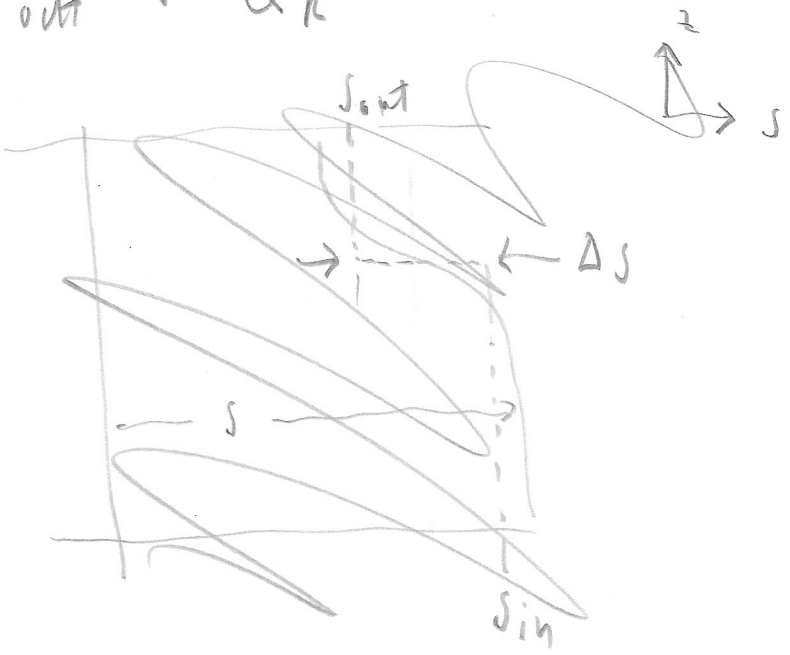
"The Knudsen Relations" (1900)

or $\quad \boxed{-Q_{out} = \frac{s_{in}}{\Delta s} Q_R + \frac{1}{\Delta s}\frac{d}{dt}\left\langle \int s\, dV \right\rangle}$

exchange flow — often $\Delta s \ll s_{in,out}$ so $Q_{in,out} \gg Q_R$ — storage

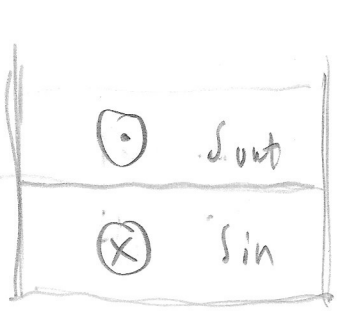
(like $[u'] \gg \bar{u}$)

From the Knudsen Relations we can estimate $Q_{in,out}$ from knowledge of $s_{in,out}$ & Q_R



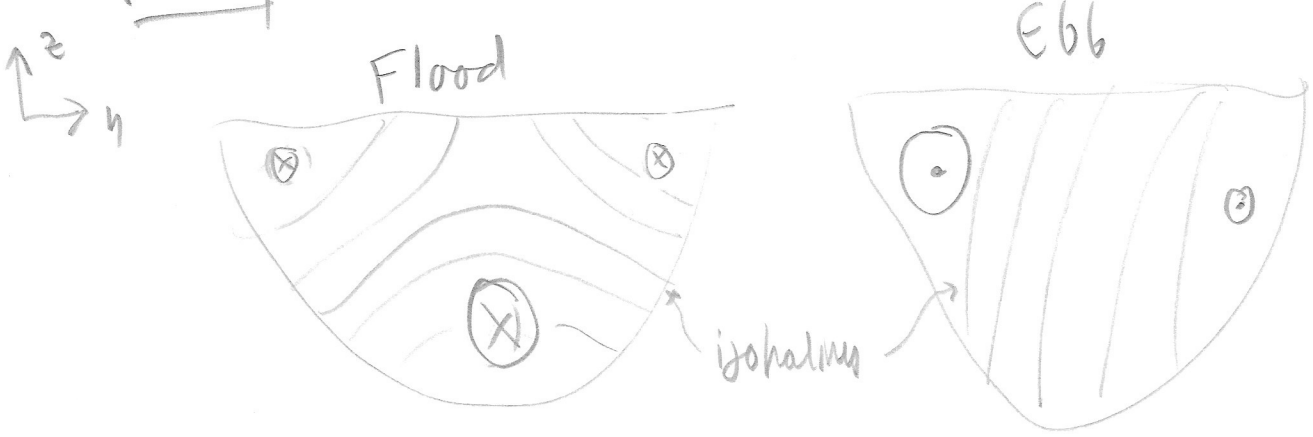
* much easier than measuring $Q_{in,out}$!

Issue: Real Estuarine salt flux through a section is not two layers:

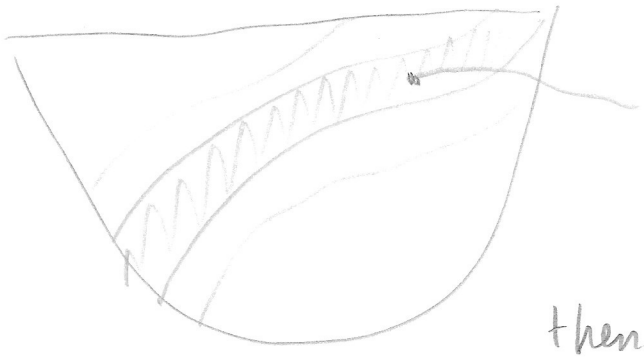


← Idealized

Reality: it is often more of a mess:



Solution: Average transport in salinity classes



$q(s)$ = transport between s and $s + \delta s$

then form $\langle q(s) \rangle$

then integrate over all salinity classes for which $\langle q(s) \rangle$ is of a given sign.

so $\int \langle q \rangle ds_{in} = Q_{in}$

Similarly $\int \langle sq \rangle ds_{in} = S_{in}$ eg.

$$\frac{\int \langle sq \rangle ds_{in}}{Q_{in}}$$

similar for Q_{out} and S_{out}

This is the "Total Exchange Flow" a TEF

MacCready (2011) JLO